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B71 06019

date: June 16, 1971

to: Distribution

from: P. G. Smith

subject: Spinning Flexible Spacecraft Wobble Motion and its Dependence on Flexibility - Case 620

ABSTRACT

By means of a simple model it is shown that two measures of damping in a spinning flexible spacecraft behave differently as spacecraft flexibility is varied; the two measures of damping are the time constant and rms response to random crew excitation. The way in which crew excitation is prescribed, that is, as forces or as motions, is found to have considerable influence on how response varies with flexibility.

Difference in behavior between time constant and rms response was observed in preparation of the parent memorandum,* and the present memorandum provides documentation of work done at that time to understand the phenomenon.

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*TM-71-1022-2, "Structural Wobble Damping of a Spinning Skylab," by P. G. Smith.

(NASA-CR-119079) SPINNING FLEXIBLE SPACECRAFT WOBBLE MOTION AND ITS DEPENDENCE ON FLEXIBILITY (Bellcomm, Inc.) 17 p

N79-72413

Unclas

00/18 12069

(NASA CR OR TMX OR AD NUMBER) (CATEGORY)

[redacted]

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MEMORANDUM FOR FILE

Introduction

The work reported herein was performed in conjunction with a parent study (Ref. 1) in order to obtain insight into the findings of that study. The findings are that two measures of system damping, namely time constant and rms response to random excitation,* behave differently as parameters, such as flexibility, of a spinning flexible spacecraft are varied. Specifically, it is shown that as appendage stiffness is increased, the time constant increases while the rms response decreases. The complexity of the dynamical model used in Ref. 1 obscures an understanding of these relationships, so a simple model is used here.

The sequel is organized as follows: The model is described and its equations of motion are given. Approximate formulas are obtained for the time constant and the rms response to random crew motion. Finally, results from the formulas are compared with results obtained from the computer program used in Ref. 1, and a discussion of the results follows.

Model and Equations of Motion

As shown in Figure 1, the model comprises three bodies, a main body B_1, and appendage B_2, and a crewman B_3. Relative to B_1, B_2 is allowed only rotation about X and B_3 is allowed only rotation about Y, the associated angles being θ and ψ, respectively. B_1 and B_2 are symmetrical about the spin axis Z, B_3 is inertially

*Both are measures of damping in the sense that they vary inversely with the amount of damping in the system.

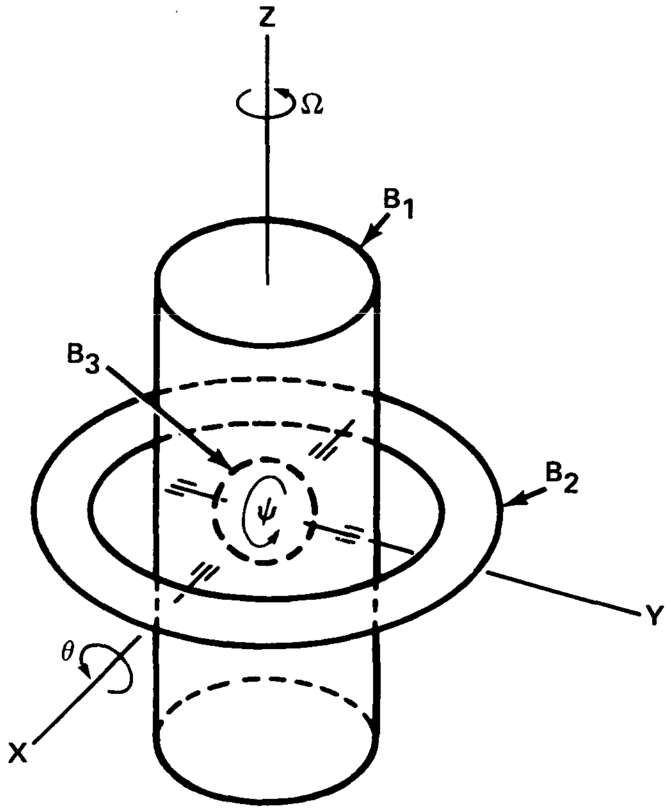

Figure 1

The Dynamical Model

spherical, and all three bodies have a common mass center. B_1 and B_2 have axial moments of inertia J_1 and J_2 and transverse moments of inertia I_1 and I_2, B_3 has moment of inertia L, Ω is the spin speed, and B_1 and B_2 are connected by a spring of constant k and a dashpot of constant c.

If ω_x, ω_y, ω_z are the angular rates of B_1, four dynamical equations can be written in the four unknown variables ω_x, ω_y, ω_z, and θ. (ψ is a prescribed function of time and hence not an unknown.) Let $\omega_x = \omega_1$, $\omega_y = \omega_2$, and $\omega_z = \Omega + \omega_3$, and linearize the equations in ω_1, ω_2, ω_3, θ, and ψ. One of the linearized equations is $\dot{\omega}_3 = 0$, and it is dropped from further consideration. The remaining equations may be written in dimensionless form as follows.

$$u_1' - K_1 u_2 + K_2 u_3'' - K_3 u_3 = \ell \psi' \tag{1}$$

$$K_1 u_1 + u_2' + (K_2 + K_3)\, u_3' = -\ell \psi'' \tag{2}$$

$$u_1' - \frac{K_3}{K_2} u_2 + u_3'' + \alpha u_3' + \left(\omega^2 - \frac{K_3}{K_2}\right) u_3 = 0 \tag{3}$$

where

$$u_1 = \omega_1/\Omega \quad , \quad u_2 = \omega_2/\Omega \quad , \quad u_3 = \theta$$

$$K_1 = \frac{I_1 - J_1 + I_2 - J_2}{I_1 + I_2 + L}$$

$$K_2 = \frac{I_2}{I_1 + I_2 + L}$$

$$K_3 = \frac{I_2 - J_2}{I_1 + I_2 + L}$$

$$\ell = \frac{L}{I_1 + I_2 + L}$$

$$\alpha = \frac{c}{I_2 \Omega} \quad , \quad \omega^2 = \frac{k}{I_2 \Omega^2}$$

The independent variable in (1) - (3) is $\tau = \Omega t$, and primes denote differentiation with respect to τ.

The parameter ω^2, which is of prime interest, may be thought of as a dimensionless appendage stiffness, or alternatively, ω may be thought of as the appendage natural frequency.*

Configuration of Interest

The phenomenon under investigation seems to be related to the spacecraft configuration. The Skylab artificial gravity configuration studied has the following characteristics:

without ballast booms B_2 the spin axis is not the axis of maximum moment of inertia,

with booms the composite vehicle does spin about its axis of maximum moment of inertia,

the boom inertias are as small as practicable, and

the crewman B_3 is much smaller than either the main body B_1 or the booms B_2.

Analytically, these characteristics are expressed as follows:

$$J_1 = (1-q)\, I_1 \quad , \qquad 0<q<1$$

$$J_1 + J_2 = (1+q)(I_1+I_2)$$

$$J_2 = 2I_2$$

$$L \ll I_2$$

Now the inertia parameters K_1, K_2, K_3 can all be expressed in terms of the single parameter q.

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*ω is analogous to the ω_B of Ref. 1.

$$K_1 = -q \tag{4}$$

$$K_2 = \frac{2q}{1+q} \tag{5}$$

$$K_3 = \frac{-2q}{1+q} \tag{6}$$

We are interested in the situation where $q<<1$.

Time Constant

The fourth-order system (1) - (3) possesses four eigenvalues, a complex pair associated with the appendage vibrations and a complex pair associated with the wobble motion of the main body. Time constant shall denote the negative reciprocal of real part of the wobble motion eigenvalues.*

The characteristic equation of (1) - (3) is

$$c_0 s^4 + c_1 s^3 + c_2 s^2 + c_3 s + c_4 = 0 \tag{7}$$

where, by use of (4) - (6),

$$c_0 = \frac{1-q}{1+q}$$

$$c_1 = \alpha$$

$$c_2 = \omega^2 + (1-q)^2$$

$$c_3 = \alpha q^2$$

$$c_4 = q^2 \, [\omega^2 - (\frac{1-q}{1+q})]$$

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*This definition is consistent with the one used in Ref. 1.

Because $\alpha << 1$, approximate analytical expressions are available for the roots of (7) (see Appendix), and the time constant is approximately

$$T = \frac{4c_0}{c_1 - |c_1c_2 - 2c_3|(c_2^2 - 4c_0c_4)^{-1/2}} \tag{8}$$

When $q << 1$, (8) yields

$$T = \frac{(\omega^2 + 1)^2}{\alpha q^2} \tag{9}$$

RMS Response

We wish to find the rms response of (1) - (3) when $\psi(t)$ (the crew motion) is a random process obtained by passing white noise through a linear filter of the form

$$H_\psi(s) = \frac{\rho^2}{\ell(s + \gamma)(s^2 + 2\zeta\rho s + \rho^2)}$$

For the system under consideration ρ is much larger than any of the other system frequencies, so it is sufficient to use

$$H_\psi(s) = \frac{1}{\ell(s + \gamma)} \tag{10}$$

The required outputs are the Euler angle rotations ϕ_x and ϕ_y relative to a spinning reference frame. The linearized kinematical equations are

$$u_1 = \phi_x' - \phi_y \tag{11}$$

$$u_2 = \phi_y' - \phi_x \tag{12}$$

Ordinarily, (11), (12) would be combined with (1) - (3) and the resulting system would be solved as a whole. The extra labor associated with solving the combined problem is so great, however, that an approximate method of handling (11) and (12) is more attractive. The method is based on the assumption that either the first terms or the second terms on the right hand side of (11) - (12) are dominant and that the nondominant ones may be neglected. By solving the problem both ways it has been found that the second terms in (11) - (12) are dominant, so we henceforth use the kinematical equations

$$\phi_x = u_2 \tag{13}$$

$$\phi_y = -u_1 \tag{14}$$

Another approximation is in order. In the course of generating data for Ref. 1 it was observed that the rms value of ϕ_x was larger than that of ϕ_y. Rather than compute rms values for both ϕ_x and ϕ_y and root sum square them, it is appropriate then to work with just the rms value of ϕ_x, which we denote ϕ_{rms}.

When (10) and (13) are combined with the Laplace transform of (1) - (3), there results a transfer function $H_\phi(s)$, which when driven by white noise yields the process $\phi_x(s)$.

$$H_\phi(s) = \frac{n_0 s^5 + n_1 s^4 + n_2 s^3 + n_3 s^2 + n_4 s + n_5}{d_0 s^5 + d_1 s^4 + d_2 s^3 + d_3 s^2 + d_4 s + d_5}$$

This is not a satisfactory transfer function because the numerator is of the same order as the denominator,* a condition that arises because of the denominator factor that was dropped

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*In order to obtain bounded mean square response the numerator must be of lower order than the denominator.

in arriving at (10). The situation can be rectified by dropping the relatively small n_0s^5 term from the numerator; for convenience, the n_1s^4 term, which is also small, will also be dropped. Thus,

$$H_\phi(s) = \frac{n_2s^3 + n_3s^2 + n_4s + n_5}{d_0s^5 + d_1s^4 + d_2s^3 + d_3s^2 + d_4s + d_5} \qquad (15)$$

The coefficients are

$$n_2 = -[\omega^2 + 1 + q\,(\tfrac{1-q}{1+q})]$$

$$n_3 = \alpha q$$

$$n_4 = q(\omega^2 + 1)$$

$$n_5 = 0$$

$$d_0 = \frac{1-q}{1+q}$$

$$d_1 = \alpha + \gamma\,(\tfrac{1-q}{1+q})$$

$$d_2 = \alpha\gamma + \omega^2 + \frac{(1-q)(1+3q)}{1+q}$$

$$d_3 = \gamma[\omega^2 + \frac{(1-q)(1+3q)}{1+q}] + \alpha q^2$$

$$d_4 = q^2[\alpha\gamma + \omega^2 - (\tfrac{1-q}{1+q})]$$

$$d_5 = \gamma q^2[\omega^2 - (\tfrac{1-q}{1+q})]$$

The mean square value of ϕ_x is

$$\phi^2_{rms} = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_\phi(ix) H_\phi(-ix)\,dx \qquad (i^2 = -1) \tag{16}$$

Ref. 2 gives the value of the integral:

$$\phi^2_{rms} = \frac{b_1(-a_2a_5+a_3a_4) + b_2(a_0a_5-a_1a_4) + b_3(-a_0a_3+a_1a_2)}{2[(a_0a_5-a_1a_4)^2 + (a_2a_5-a_3a_4)(-a_0a_3+a_1a_2)]} \tag{17}$$

where

$$a_0=d_0, \quad a_1=d_1, \quad a_2=-d_2, \quad a_3=-d_3, \quad a_4=d_4, \quad a_5=d_5$$

$$b_1=n_2^2, \quad b_2=n_3^2-2n_2n_4, \quad b_3=n_4^2$$

A Routh-Hurwitz stability analysis of characteristic equation (7) gives rise to the stability condition

$$\omega^2 > \frac{1-q}{1+q} \tag{18}$$

Therefore, we are interested in reducing (17) to a simpler formula valid when ω^2 is on the order of 10^0 and α,γ,q are all small compared to unity. Such a formula is

$$\phi^2_{rms} = \frac{(\omega^2+1)^3}{4\alpha[q^2(\omega^2-1+2q) + \gamma^2(\omega^2+1)]} \tag{19}$$

Discussion

Approximate formulas (9) and (19) give the time constant and rms response, respectively, for a simple model of a spinning flexible spacecraft. The formulas are plotted in Figures 2 and 3, respectively, (solid lines) for $\alpha=q=10^{-2}$ and for two values of γ, $\gamma=10^{-2}$ and $\gamma=10^{-3}$. Observe that for $\gamma=10^{-3}$ and small values of ω^2 (appendage stiffness), the phenomenon of interest does occur, namely, the rms response decreases as the time constant increases.

The dashed lines in Figures 2 and 3 are obtained from the computer program written in connection with Ref. 1. The program input is chosen to represent what seem to be the essential characteristics of the simple model. As can be seen from the figures, agreement between the models is good in regard to the time constant and the $\gamma=10^{-2}$ response, and the $\gamma=10^{-3}$ response agreement is satisfactory, taking into account the essential differences between the models and the approximations on which (19) is based.

The difference in behavior between Figures 2 and 3 seems to be due to the way in which the crew excitation is prescribed. Clearly, crew motion parameter γ is important, for when $\gamma=10^{-2}$ both time constant and rms response increase monotonically with ω^2.

The form in which crew excitation is applied is also important. Crew disturbance data are given in terms of forces and moments applied to the spacecraft by the crewman (Ref. 3). In Ref. 1, however, the crew disturbance data are divided by the crewman's inertia and integrated twice to give crew motions, and it is crew motions rather than forces and moments that constitute the crew disturbances. In order to keep the two models similar, the simple model treated here is handled in the same way. But the simple model has also been tried with prescribed crew moments rather than the crew motions $\psi(t)$, and with prescribed crew moments the response <u>increases</u> monotonically with ω^2. The difference in the two approaches is manifested in the term on the right hand side of (1) that is present when crew motion is prescribed but absent when crew moment is prescribed.

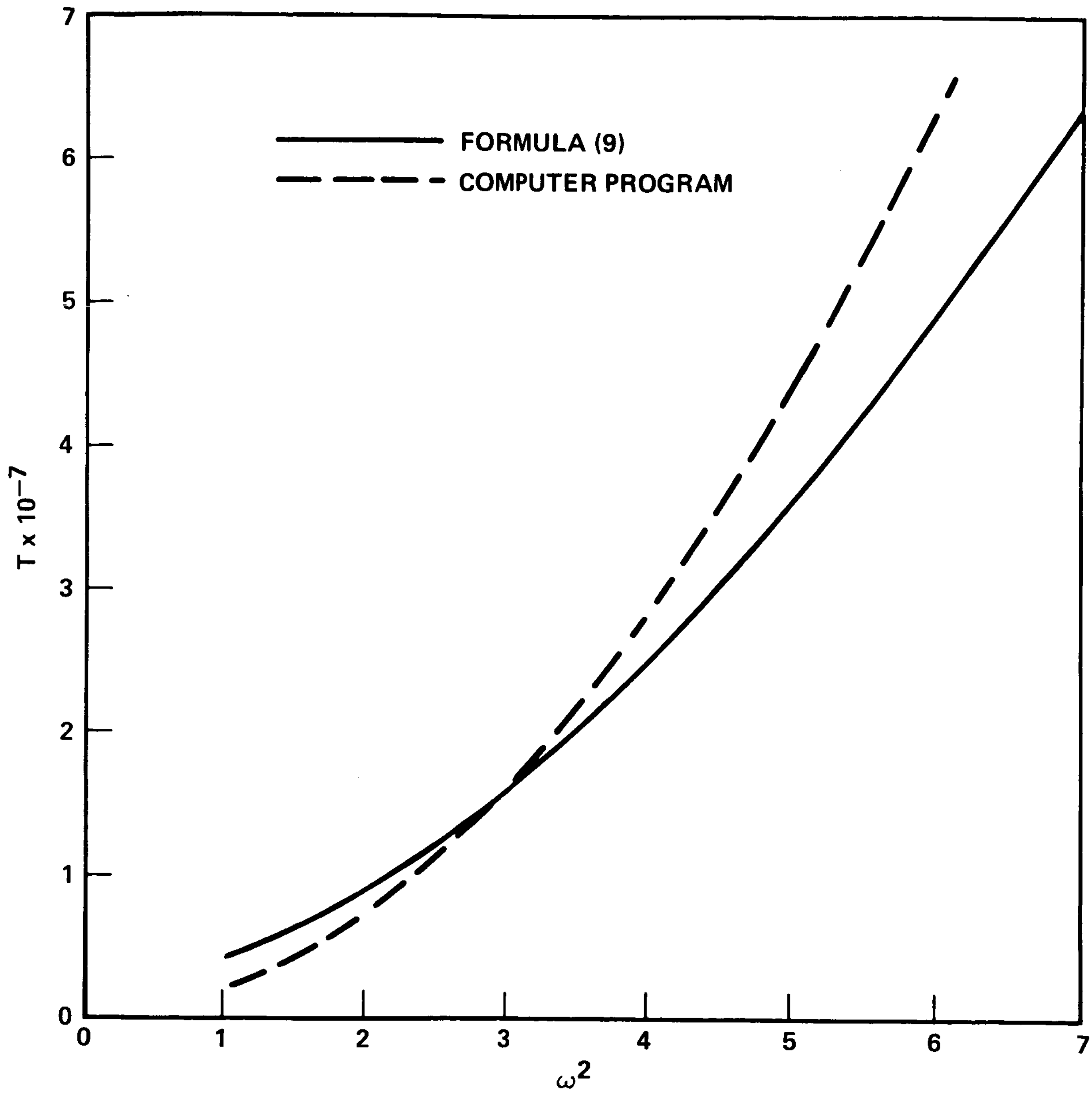

Figure 2
Time Constant vs. Appendage Stiffness (Independent of γ)

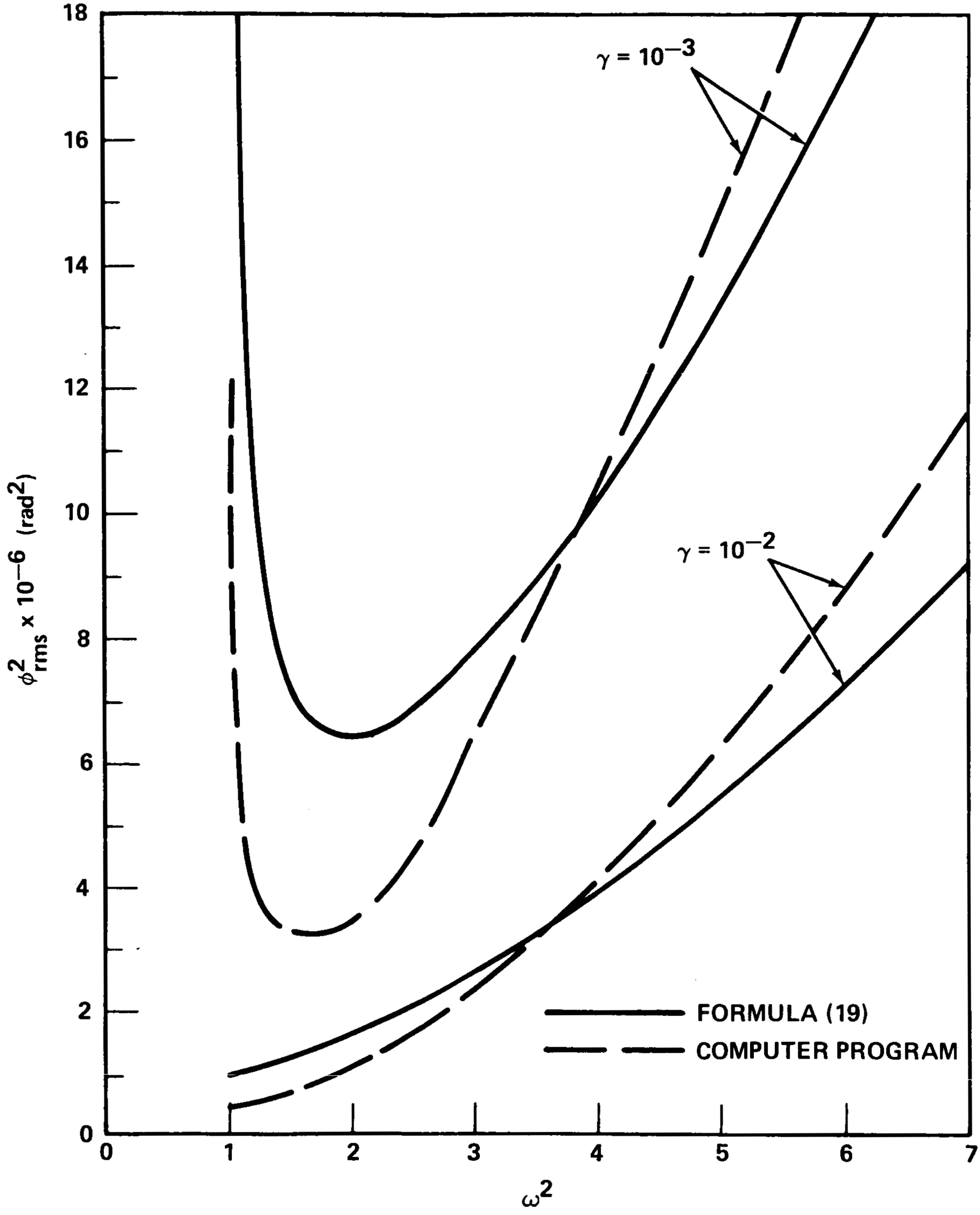

Figure 3

RMS Response vs. Appendage Stiffness

The question arises as to whether it is more realistic to prescribe crew motions or crew moments. For a spacecraft whose gross motion is negligible (such as an inertially oriented spacecraft) the two approaches are essentially the same. For the Skylab artificial gravity configuration, the inertia forces induced by the spacecraft's spin will, in the writer's opinion, be sufficiently small that a crewman will be able to carry out his desired motions in performing specific tasks, the same motions he would have performed had the spacecraft not been spinning. In this case, motions rather than moments should be prescribed. Only when inertia forces become so large that the crewman has difficulty overcoming them should moments be prescribed instead.

Conclusions

The simple model treated herein does exhibit the phenomenon in question, namely that time constant, T, increases and rms response, ϕ_{rms}, decreases as appendage stiffness, ω^2, is increased. The decrease occurs only over a range of ω^2, and the decrease is attributable to the form of crew excitation used, namely prescribed crew motions with a small parameter γ. This form of crew excitation, which seems realistic for the problem at hand, is the one used in Ref. 1.

Results from the computer program used in Ref. 1 agree well with results derived herein for the simple model.

P. G. Smith

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P. G. Smith

Attachments
  Appendix
  References

APPENDIX

Approximate Roots of a Lightly Damped Quartic Equation

We desire approximate formulas for the roots of

$$p_0 x^4 + p_1 x^3 + p_2 x^2 + p_3 x + p_4 = 0 \qquad \text{(A-1)}$$

when $p_0, \ldots, p_4$ are all positive, $p_0 p_4 < p_2^2/4$, and when p_1, p_3 are "small" compared to p_0, p_2, p_4 (this is the lightly damped condition). The approach is as follows: because of the lightly damped condition, the roots will be near the imaginary axis, so neglect p_1 and p_3 terms and find roots of $p_0 x^4 + p_2 x^2 + p_4 = 0$; assuming that the roots of (A-1) are near the purely imaginary roots just found, compute real and imaginary perturbations by means of linear perturbation equations.

When $p_1 = p_3 = 0$

$$x = \frac{-p_2 \pm \sqrt{p_2^2 - 4p_0 p_4}}{2p_0} \equiv -h^2 \qquad \text{(A-2)}$$

Let the roots of (A-1) be

$$x = f + i\ (h+g) \qquad \text{(A-3)}$$

where $i^2 = -1$, and assume that $|f| \ll |h|$ and $|g| \ll |h|$. Substitute (A-3) into (A-1), separate the resulting equation into equations in the real and imaginary parts, linearize in f and g, and make use of the identity

$$p_0 h^4 - p_2 h^2 + p_4 = 0$$

The results are

$$(3p_1h^2 - p_3)f - (4p_0h^3 - 2p_2h)\,g = 0$$

$$(4p_0h^3 - 2p_2h)\,f + (3p_1h^2 - p_3)g = p_3h - p_1h^3$$

We desire the time constant

$$T = \frac{-1}{f} = \frac{16h^2\left(p_0h^2 - \frac{p_2}{2}\right)^2 + (3p_1h^2 - p_3)^2}{4h^2\left(p_0h^2 - \frac{p_2}{2}\right)(p_1h^2 - p_3)}$$

When p_1 and p_3 are small

$$T = \frac{4p_0h^2 - 2p_2}{p_1h^2 - p_3}$$

and h^2 can be removed by use of (A-2):

$$T = \frac{4p_0}{p_1 \mp (p_1p_2 - 2p_3)(p_2^2 - 4p_0p_4)^{-1/2}}$$

The <u>system</u> time constant is the larger of these two values, namely

$$T = \frac{4p_0}{p_1 - |p_1p_2 - 2p_3|\,(p_2^2 - 4p_0p_4)^{-1/2}} \qquad \text{(A-4)}$$

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